

Viscous Chaplygin Gas Models as a Spherical Top-Hat Collapsing Fluids

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Abstract

We study the spherical top-hat collapse in Einstein gravity and loop quantum cosmology by taking the non-linear evolution of viscous modified variable chaplygin gas and viscous generalized cosmic chaplygin gas. We calculate the equation of state parameter, square speed of sound, perturbed equation of state parameter, perturbed square speed of sound, density contrast and divergence of peculiar velocity in perturbed region and discussed their behavior. It is observed that both chaplygin gas models support the spherical collapse in Einstein as well as loop quantum cosmology because density contrast remains positive in both cases and the perturbed equation of state parameter remains positive at the present epoch as well as near future. It is remarked here that these parameters provide the consistence results for both chaplygin gas models in both gravities.

Keywords: Top-hat collapse; Chaplygin gas models; Einstein and Loop quantum cosmology; Density contrast.

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1 Introduction

The discovery of accelerating expansion of the universe is a milestone for cosmology. This acceleration is generally believed to be caused by the vacuum energy or exotic matter called "dark energy" (DE) [1, 2]. This exotic matter has positive energy density and strong negative pressure which can be represented by an equation of state (EoS) parameter $\omega = \frac{p}{\rho} < \frac{-1}{3}$. The simplest candidate of DE is cosmological constant (Λ) having EoS parameter $\omega = -1$ [3]. Other candidates are quintessence ($-1 < \omega < \frac{-1}{3}$) [4] and phantom ($\omega < -1$) [5]. In the universe, the contribution of DE in energy density is about 73% and an unknown form of matter called "dark matter" (DM) is about 23%. The remaining 4% of the energy density corresponds to ordinary known matter. DM (in its cold version) is a dust-like fluid with no pressure. It is attractive in nature and can not be seen by a telescope. It does not absorb or emit light or any gravitational waves. The existence of this type of matter has been proved by gravitational effect on visible matter and gravitational lensing of background radiation.

The Chaplygin gas (CG) has been presented which possesses the unified picture of DE and DM. It was introduced by Chaplygin who studied it in hydrodynamical context [6]. This gas was not consistent with angular power spectrum of the cosmic microwave background (CMB) or the angular distance scale of the baryon acoustic oscillations (BAO) [7, 8]. Therefore, an extension of CG model named as generalized CG (GCG) has been proposed. This model corresponds to almost dust ($p = 0$) at high density which does not agree completely with the universe. Then GCG is extended to modified CG (MGCG). The MGCG is more appropriate choice to have constant negative pressure at low energy density and high pressure at high density. The viscous CG model is the suitable candidate of unified DE and cold DM as a unique imperfect fluid [9, 10]. It has bulk viscosity with negative pressure suggested by observations [9, 10]. The Generalized Cosmic Chaplygin Gas (GCCG) is another form of DE having the consequence of an accelerating phase of universe, which was introduced by Gonzalez-Diaz [11]. This model is stable and free from unphysical behaviour even when the vacuum fluid satisfies the phantom energy condition.

The "cosmological collapse", a mechanism proposed by physicists predicts that the universe will soon stop expanding and collapse in itself. The spherical collapse (SC), introduced by Gunn and Gott [12] provides a way to glimpse into the nonlinear regime of perturbation theory. It explains how

a small spherical patch of homogeneous overdensity forms a bound system via gravitation instability [13]. It describes the evolution of a spherically symmetric perturbation embedded in a static, expanding or collapsing homogeneous background. One assumes a spherical top hat profile for the perturbed region, i.e. a spherically symmetric perturbation in some region of space with constant density [14]. The assumption of a top hat profile leads to the SC model as the uniformity of the perturbation is maintained throughout the collapse, making its evolution only time dependent. As a consequence, we do not need to worry about gradients inside the perturbed region. Thus, the STHC identifies the evolution of a homogeneous mini-universe inside a larger homogeneous universe.

Specifically, it is suggested that the non-linear effects of GCG can not be ignored because it can produce a back-reaction in the background dynamics, leads to crucial constraints on the validity of linear theory as soon as the first scales become non-linear. In [15], the authors have studied the non-linear evolution of dark matter and dark energy by taking CG model, using generalizations of the spherical model that incorporated effects of the acoustic horizon. An interesting phenomenon was found there: a fraction of the CG condensated and never reached a stage where its properties changed from dark-matter-like to dark-energy-like. A fully non-linear analysis is a cumbersome task usually handled by hydrodynamical/Nbody numerical codes (see e.g. [14, 16]).

Moreover, some works have been done by assuming spherically symmetric perturbation in some region of space with constant density as a spherical top-hat profile for the perturbed region. Fabris et al. [17] studied the evolution of density perturbation in a universe dominated by CG. Their model gave the required density contrast observed in large scale structures of universe in Newtonian approach. Carturan and Finelli [18] conducted same investigations for GCG. Hence, Fernandes et al. [14] studied the evolution of density perturbation in a universe dominated by CG. Their model gave the required density contrast observed in large scale structures of universe in Newtonian approach. Carturan and Finelli [18] conducted same investigations for GCG. Cramés et al. [19] investigated the STHC model using viscous GCG (VGCG). Li and Xu [9, 10] have extended to above work by adding bulk viscosity in GCG model. They analyzed effect of bulk viscosity on structure formations of the GCG model having spherically symmetric perturbations.

Recently, Karbasi and Razmi [20] have discussed the STHC in the presence of MCG. Also, Ujjal and Mubasher [21] have analyzed the STHC sce-

nario in the presence of viscous MCG in Einstein and loop quantum gravities. Motivated by these works, we study the STHC in the presence of viscous modified variable CG (VMVCG) and viscous GCCG (VGCCG) in Einstein as well as loop quantum gravities. We organize our paper as follows: In section **2**, we give a discussion of VMVCG and corresponding STHC scenario in both gravities. Section **3** provides STHC of VGCCG. We conclude the results in Section **4**.

2 Viscous Modified Variable Chaplygin Model

We consider the flat FRW universe as

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2], \quad (1)$$

where $a(t)$ is the scale factor of the universe. The Einstein field equations are given by

$$H^2 = \frac{8\pi G}{3}\rho, \quad \dot{H} = -4\pi G(\rho + p), \quad (2)$$

Next, we take $8\pi G = 1$. The CG acts as a pressureless fluid for small values of the scale factor and tends to accelerated expansion for large values of the scale factor. The EoS for this model has following form

$$p_d = -\frac{B}{\rho},$$

where B is a positive constant. The EoS parameter for CG has been generalized to the form [22]-[24].

$$p_d = -\frac{B}{\rho^\alpha},$$

with $0 < \alpha \leq 1$. The EoS of CG has an interesting connection with the D-branes which are expressed via Nambu-Goto action [17]. It also enjoys connections with the Newtonian hydrodynamical equations. Further the Eddington-Born-Infeld model can be seen as an affine connection version for the CG approach [25]. The CG has been extensively studied within the unified DE-DM models. The CG was extended to MVCG with following EoS

$$p_d = \tilde{A}\rho - \frac{B(a)}{\rho^\alpha}, \quad (3)$$

where $B(a) = B_0 a^{-m}$ with m a non-negative integer and \tilde{A} is a constant constrained by the astrophysical data. The special case $\tilde{A} = \frac{1}{3}$ is the best fitted value to describe evolution of the universe from radiation regime to Λ -cold DM regime.

The MVCG satisfactorily accommodates an accelerating phase as well as matter dominated phase of the universe. It is also consistent with the observational studies dealing with the large scale structure [26]. We assume here that the spacetime is filled with only one component fluid having a bulk viscosity. This component is defined in terms of effective pressure p as follows

$$p = p_d + \Pi, \quad (4)$$

which is the sum of the equilibrium pressure p_d and the bulk pressure $\Pi = -\xi_{;\gamma}^{\gamma}$ where u^{γ} is the four velocity of the fluid and ξ represents the coefficient of bulk viscosity (which is a function of energy density). The first attempts at creating a viscosity theory of relativistic fluids were executed by Eckart [27] and Landau and Lifshitz [28]. They considered only a first-order deviation from equilibrium. The bulk viscous pressure p_d is represented by Eckart's expression which is proportional to the Hubble parameter H . The proportionality factor identified as the bulk viscosity coefficient $\xi = \xi_0 \rho^e$ where ξ_0 and e are constants. For simplicity, choosing $e = \frac{1}{2}$, Π can be written as [7],

$$\Pi = -3\xi_0 H \sqrt{\rho},$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter with dot representing time derivative.

By combining Eqs.(3)-(5), we obtain VMVCG

$$p = \tilde{A}\rho - \frac{B_0 a^{-m}}{\rho^{\alpha}} - 3\xi_0 H \sqrt{\rho}. \quad (5)$$

The conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (6)$$

By inserting Eq.(5) in (6), we get the following solution

$$\rho = \left(\frac{3B_0(1+\alpha)}{3(1+\alpha)(1+\tilde{A}-\sqrt{3}\xi_0) - m} a^m + \frac{C}{a^{3(1+\alpha)(1+\tilde{A}-\sqrt{3}\xi_0)}} \right)^{\frac{1}{\alpha+1}}. \quad (7)$$

In terms of redshift, i.e. using $a = \frac{1}{1+z}$, we obtain

$$\rho(z) = \rho_0 \left(\frac{3B_0(1+\alpha)(1+z)^m}{3(1+\alpha)(1+\tilde{A}-\sqrt{3}\xi_0) - m} + C(1+z)^{3(1+\alpha)(1+\tilde{A}-\sqrt{3}\xi_0)} \right)^{\frac{1}{\alpha+1}} \quad (8)$$

By inserting the above expression in field equation, we will get the Hubble parameter as follows

$$H(z) = H_0 \left[\left(\frac{3B_0(1+\alpha)(1+z)^m}{3(\alpha+1)(1+\tilde{A}-\sqrt{3}\xi_0)-m} C(1+z)^{3(\alpha+1)(1+\tilde{A}-\sqrt{3}\xi_0)} \right)^{\frac{1}{\alpha+1}} \times \Omega_0 \right]^{\frac{1}{2}}. \quad (9)$$

The EoS parameter becomes

$$\omega = \frac{p}{\rho} = \tilde{A} - \frac{B_0 a^{-m}}{\rho^{\alpha+1}} - \frac{3\xi_0 H}{\sqrt{\rho}}. \quad (10)$$

The adiabatic sound speed can be defined as follows

$$c_s^2 = \frac{dp}{d\rho}. \quad (11)$$

For VMVCG, this parameter can be obtained by using Eqs. (5)-(9).

2.1 STHC for VMVCG in Einstein Gravity

Following the assumption of a top-hat profile, the density perturbation is uniform throughout the collapse. In this case the evolution of perturbation is only time-dependent. In STHC model, the background evolution equation are in following form

$$\dot{\rho} = -3H(\rho + p), \quad \frac{\ddot{a}}{a} = \frac{-4\pi G}{3} \sum_i (\rho_i + p_i).$$

The perturbed quantities ρ_c and p_c are related to their background counterparts by $\rho_c = \rho + \delta\rho$ and $p_c = p + \delta p$. Applying the perturbations, the EoS for VMVCG becomes

$$\omega_c = \frac{\tilde{A} - \frac{B_0 a^{-m}}{\rho^{\alpha+1}} - \frac{3\xi_0 H}{\sqrt{\rho}} + \delta c_e^2}{1 + \delta}. \quad (12)$$

The perturbed equation of sound speed leads to

$$c_e^2 = \frac{p_c - \rho \tilde{A} + \frac{B_0 a^{-m}}{\rho^\alpha} + 3\xi_0 H \sqrt{\rho}}{\rho_c - \rho},$$

which implies

$$c_e^2 = \dot{A} - \frac{B_0(1+z)^{-m}(1-(1+\delta)^\alpha)}{\delta\rho^{1+\alpha}(1+\delta)^\alpha} - \frac{3}{\delta\sqrt{\rho}}H\xi_0(\sqrt{1+\delta}-1). \quad (13)$$

The basic equations about the density contrast and divergence of peculiar velocity in perturbed region are presented in the **Appendix**. Hence, by following the procedure of [20, 21], we can get density contrast and divergence of peculiar velocity for VMVCG in the following for Einstein gravity

$$\begin{aligned} \frac{d\delta}{dz} &= \frac{3}{1+z} \left(\tilde{A} - \frac{B_0(1+z)^{-m}(1-(1+\delta)^\alpha)}{\delta\rho^{1+\alpha}(1+\delta)^\alpha} - \frac{3}{\delta\sqrt{\rho}}H\xi_0(\sqrt{1+\delta}-1) \right. \\ &\quad \left. - \omega \right) \delta - \left(1 + \omega + \left(1 + \left(\tilde{A} - \frac{B_0(1+z)^{-m}(1-(1+\delta)^\alpha)}{\delta\rho^{1+\alpha}(1+\delta)^\alpha} - \frac{3}{\delta\sqrt{\rho}}H\xi_0 \right. \right. \right. \\ &\quad \left. \left. \left. \times (\sqrt{1+\delta}-1) \right) \right) \delta \right) \frac{\theta}{H}, \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d\theta}{dz} &= \frac{\theta}{1+z} + \frac{\theta^2}{3H(z)} + \frac{3H(z)}{2(1+z)^2} \left(1 + 3 \left(\tilde{A} - \frac{B_0(1+z)^{-m}(1-(1+\delta)^\alpha)}{\delta\rho^{1+\alpha}(1+\delta)^\alpha} \right. \right. \\ &\quad \left. \left. - \frac{3}{\delta\sqrt{\rho}}H\xi_0(\sqrt{1+\delta}-1) \right) \right) \delta\Omega. \end{aligned} \quad (15)$$

2.2 STHC for VMVCG in Loop Quantum Cosmology

The LQC is a quantization of symmetry reduced spactetime. Some phenomenon like predictions of cosmic inflation in the early universe [30], late time cosmic acceleration [31] and primordial gravitational waves [32] have also explored in this gravity. In this gravity, the cosmological perturbation theory has also investigated [33]. The LQC possesses the properties of non-perturbative and background independent quantization of gravity [34]-[39]. Various DE models have been investigated in this gravity [40, 41]. Jamil et al. [42] have investigated the cosmic coincidence problem by assuming the MCG coupled to DM. It has also been found that the future singularity appearing in the standard FRW cosmology can be avoided by loop quantum effects [43]. Chakraborty et al. [44] have tested MCG in LQC.

The modified Einstein's field equations in LQC for FRW metric are given by

$$H^2 = \frac{8\pi G\rho}{3} \left(1 - \frac{\rho}{\rho_1} \right), \quad (16)$$

$$\dot{H} = -4\pi G(\rho + p) \left(1 - \frac{2\rho}{\rho_1}\right), \quad (17)$$

where $\rho_1 = \frac{\sqrt{3}}{16\pi^2\gamma^3 G^2 h}$ is called the critical loop quantum density, γ is the dimensionless Barbero-Immirzi parameter.

In this case, the Hubble parameter is obtained as

$$\begin{aligned} H(z) &= H_0 \left[\Omega_0 \frac{3B_0(1+\alpha)(1+z)^m}{3(1+\alpha)(1+\tilde{A}-3\xi_0)-m} + c(1+z)^{3(1+\alpha)(1+\tilde{A}-3\xi_0)} \right]^{\frac{1}{1+\alpha}} \\ &\times \left[1 - \frac{\rho_0}{\rho_1} \left(\frac{3B_0(1+\alpha)(1+z)^m}{3(1+\alpha)(1+\tilde{A}-3\xi_0)-m} + c(1+z)^{3(1+\alpha)(1+\tilde{A}-3\xi_0)} \right. \right. \\ &\quad \left. \left. - 3\xi_0 \right)^{\frac{1}{1+\alpha}} \right]^{\frac{1}{2}}. \end{aligned} \quad (18)$$

In this case, the EoS parameter (10) becomes

$$\omega = \tilde{A} - \frac{B_0 a^{-m}}{\rho^{\alpha+1}} - \frac{3\xi_0 H_0 \sqrt{\frac{\Omega_0 \rho}{\rho_0} \left(1 - \frac{\rho}{\rho_1}\right)}}{\sqrt{\rho}}. \quad (19)$$

The adiabatic sound speed becomes

$$c_s^2 = \tilde{A} - \frac{\alpha B_0 a^{-m}}{\rho^{\alpha+1}} - \frac{3\xi_0}{2\sqrt{\rho}} H_0 \sqrt{\frac{\Omega_0 \rho}{\rho_0} \left(1 - \frac{\rho}{\rho_1}\right)}, \quad (20)$$

the perturbed EoS (14) turns out to be

$$\omega_c = \frac{1}{1+\delta} \left(\tilde{A} - \frac{B_0 a^{-m}}{\rho^{\alpha+1}} - \frac{3\xi_0 H}{\sqrt{\rho}} + \delta c_e^2 \right). \quad (21)$$

The perturbed equation of sound speed in terms of redshift takes the form

$$\begin{aligned} c_e^2 &= \dot{A} - \frac{B_0(1+z)^{-m}(1-(1+\delta)^\alpha)}{\delta\rho(1+\alpha)(1+\delta)^\alpha} - \frac{3\xi_0}{\delta\sqrt{\rho}} H_0 \sqrt{\frac{\Omega_0 \rho}{\rho_0} \left(1 - \frac{\rho}{\rho_1}\right)} \\ &\times (\sqrt{1+\delta} - 1). \end{aligned} \quad (22)$$

The dynamical equations of density contrast δ remains the same whereas equation for θ for MVGCG in LQC becomes

$$\frac{d\theta}{dz} = \frac{\theta}{1+z} + \frac{\theta^2}{3H(z)} + \frac{3H(z)}{2(1+z)^2} \left(1 + 3 \left(\tilde{A} - \frac{B_0(1+z)^{-m}(1-(1+\delta)^\alpha)}{\delta\rho(1+\alpha)(1+\delta)^\alpha} \right) \right)$$

$$\begin{aligned}
& - \frac{3}{\delta\sqrt{\rho}} H \xi_0 (\sqrt{1+\delta} - 1) \Big) \Big) \delta \Omega \Big((1 + 3c_e^2) - \frac{6\Omega H^2}{\rho_1} \Big(3(1+\delta)(1+3c_e^2) \\
& + (3\omega - \delta + 1) \Big) \Big). \tag{23}
\end{aligned}$$

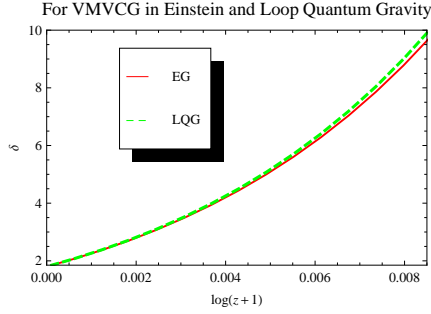


Figure 1: Plot of δ versus $\log(1+z)$.

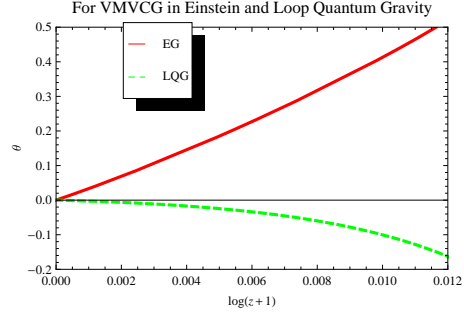


Figure 2: Plot of θ versus $\log(1+z)$.

We plot the time varying parameters δ , θ , c_e^2 , c_s^2 , ω_c and ω versus $\log(1+z)$ for VMVCG in Einstein gravity and loop quantum cosmology as shown in Figures 1-6. Here, we use $\Omega = \frac{\rho}{3H^2}$. However, other constants are $\tilde{A} = 1.2$, $B_0 = 10.2$, $C = 0.2$, $\xi_0 = 0.01$, $\alpha = 1.5$, $H_0 = 72$, $\rho_0 = 0.23$, $\rho_1 = 2.01$ and $\Omega_0 = 1$. We have chosen the well-known initial condition as the present value of redshift parameter, i.e., $z = 0$ which leads to $\log[1+z] \rightarrow 0$ in our case in all plots. Figure 1 shows the evolution of density perturbation and possesses the increasing behavior with the passage of time. On the other hand, the other perturbed quantity θ exhibits the increasing behavior with the passage of time in Einstein gravity while shows decreasing behavior in loop quantum cosmology (Figure 2). Figure 3 indicates that the squared speed of sound remains positive which leads to the stability of the model at the present as well as later epoch. In Figure 4, the perturbed squared speed of sound shows increasing/decreasing behavior for Einstein/Loop quantum cosmology respectively, while approaches to a positive constant after some interval of time. However, this remains positive throughout cosmic time which is consistence behavior with usual squared speed of sound. Figure 5 (the perturbed EoS) depicts matter dominated era of the universe at initial epoch while goes to phantom era at the later epoch by crossing quintessence

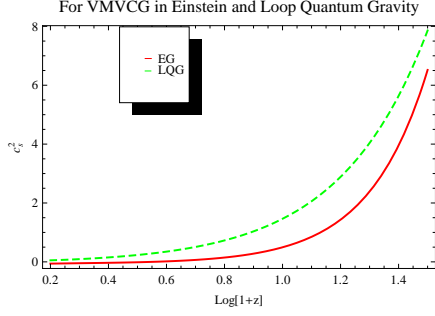


Figure 3: Plot of c_s^2 versus $\log(1+z)$.

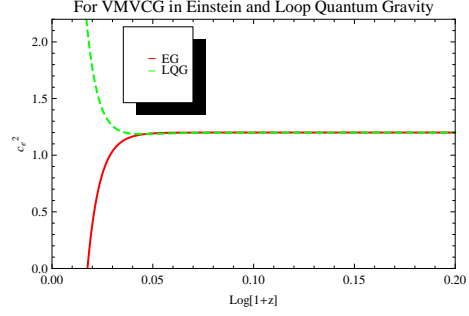


Figure 4: Plot of c_e^2 versus $\log(1+z)$.

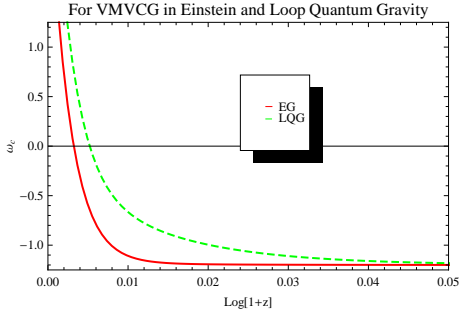


Figure 5: Plot of ω_c versus $\log(1+z)$.

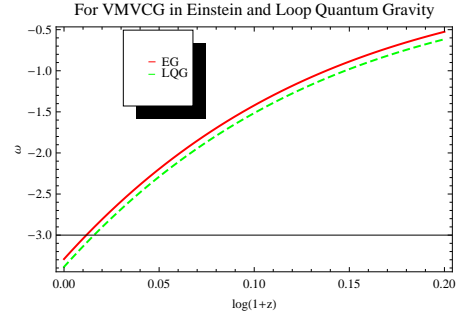


Figure 6: Plot of ω versus $\log(1+z)$.

as well as Λ CDM limit. On the other hand, the usual EoS parameter shows phantom-like universe at the present epoch while goes towards quintessence region of the universe at the later epoch (Figure 6).

3 Viscous Generalized Cosmic Chaplygin Gas

According to earlier studies related to DE corresponding to phantom era, big rip was the final destination as the time derivative of scale factor goes to infinity in finite time. By using GCCG model, big rip singularity can be avoided. The EoS for GCCG has following form

$$p_d = -\rho^{-\alpha} \left(C + (\rho^{1+\alpha} - C)^{-d} \right), \quad C = \frac{A'}{(1+\omega)} - 1$$

A' takes either positive or negative constant value, $-l < d < 0$ and $l > 1$. The EoS reduces to that of current Chaplygin unified models for DM and DE in the limit $\omega \rightarrow 0$ and satisfies the conditions:

- It behaves like de Sitter fluid with $\omega = -1$.
- It becomes $p = \omega\rho$ as chaplygin parameter $A' \rightarrow 0$.
- It reduces to the EoS of current chaplygin unified DM model at high energy density.
- The evolution of density perturbations derived from the chosen EoS becomes free from the pathological behavior of the matter power spectrum for physically reasonable values of the involved parameter at late time. This EoS shows a dust era in the past and Λ CDM in future.

After adding bulk viscosity, above equation becomes

$$p_d = -\rho^{-\alpha}[C + (\rho^{1+\alpha} - C)^{-d}] - 3\xi_0 H \sqrt{\rho}.$$

The solution of conservation equation after substituting above equation is given by

$$\rho = \left(C + \left(\frac{1}{1 - 3\sqrt{3}\xi_0} + \frac{B}{a^{3(1+\alpha)(1+d)(1-3\sqrt{3}\xi_0)}} \right)^{\frac{1}{1+d}} \right)^{\frac{1}{1+\alpha}}. \quad (24)$$

This expression reduces to the following form in terms of redshift,

$$\rho = \left(C + \left(\frac{1}{1 - 3\sqrt{3}\xi_0} + B(1+z)^{3(1+\alpha)(1+d)(1-3\sqrt{3}\xi_0)} \right)^{\frac{1}{1+d}} \right)^{\frac{1}{1+\alpha}}.$$

The Hubble parameter turns out to be

$$\begin{aligned} H(z) &= H_0 \left(\left(C + \left(\frac{1}{1 - 3\sqrt{3}\xi_0} + B(1+z)^{3(1+\alpha)(1+d)(1-3\sqrt{3}\xi_0)} \right)^{\frac{1}{1+d}} \right)^{\frac{1}{1+\alpha}} \right. \\ &\quad \times \left. \Omega_0 \right)^{\frac{1}{2}}. \end{aligned} \quad (25)$$

In this case, the EoS parameter becomes

$$\omega = -\rho^{-(\alpha+1)}[C + (\rho^{1+\alpha} - C)^{-d}] - \frac{3\xi_0 H}{\sqrt{\rho}}. \quad (26)$$

3.1 STHC for VGCCG in Einstein Gravity

For VGCCG, the perturbed EoS parameters takes the following form

$$\omega_c = \frac{1}{1+\delta} \left(-\rho^{-(\alpha+1)} (C + (\rho^{1+\alpha} - C)^{-d}) - \frac{3\xi_0 H}{\sqrt{\rho}} + c_e^2 \delta \right). \quad (27)$$

Similarly, the perturbed equation of sound speed turns out to be

$$\begin{aligned} c_e^2 &= \frac{1}{\rho\delta} C (\rho^{-\alpha} - \rho(1+\delta)^{-\alpha}) - 3\xi_0 H \sqrt{\rho} (1 - \sqrt{1+\delta}) + \rho^{-\alpha} (\rho^{1+\alpha} - C)^{-d} \\ &\quad - \rho^{-\alpha} (1+\delta)^{-\alpha} (\rho^{1+\alpha} (1+\delta)^{1+\alpha})^{-d}. \end{aligned} \quad (28)$$

In this scenario, Eqs.(34) and (35) leads to

$$\begin{aligned} \frac{d\delta}{dz} &= \frac{3}{1+z} \left(\left(\frac{1}{\rho\delta} C (\rho^{-\alpha} - \rho(1+\delta)^{-\alpha}) - 3\xi_0 H \sqrt{\rho} (1 - \sqrt{1+\delta}) \right. \right. \\ &\quad \left. \left. + \rho^{-\alpha} (\rho^{1+\alpha} - C)^{-d} - \rho^{-\alpha} (1+\delta)^{-\alpha} [\rho^{1+\alpha} (1+\delta)^{1+\alpha}]^{-d} \right) - \omega \right) \delta \\ &\quad - \left(1 + \omega + \left(1 + \left(\frac{1}{\rho\delta} C (\rho^{-\alpha} - \rho(1+\delta)^{-\alpha}) - 3\xi_0 H \sqrt{\rho} (1 - \sqrt{1+\delta}) \right. \right. \right. \\ &\quad \left. \left. \left. + \rho^{-\alpha} (\rho^{1+\alpha} - C)^{-d} - \rho^{-\alpha} (1+\delta)^{-\alpha} (\rho^{1+\alpha} (1+\delta)^{1+\alpha})^{-d} \right) \right) \frac{\theta}{H(z)} \right. \\ \frac{d\theta}{dz} &= \frac{\theta}{1+z} + \frac{\theta^2}{3H(z)} + \frac{3H(z)}{2(1+z)^2} \left(1 + 3 \left(\frac{1}{\rho\delta} C (\rho^{-\alpha} - \rho(1+\delta)^{-\alpha}) \right. \right. \\ &\quad \left. \left. - 3\xi_0 H \sqrt{\rho} (1 - \sqrt{1+\delta}) + \rho^{-\alpha} (\rho^{1+\alpha} - C)^{-d} - \rho^{-\alpha} (1+\delta)^{-\alpha} \right. \right. \\ &\quad \left. \left. \left. + \delta)^{-\alpha} (\rho^{1+\alpha} (1+\delta)^{1+\alpha})^{-d} \right) \right) \delta \Omega. \end{aligned}$$

3.2 STHC for VGCCG in Loop Quantum Cosmology

For VGCCG model in LQC, the Hubble parameter is

$$\begin{aligned} H(z) &= H_0 \left[\Omega_0 \left(C + \left(\frac{1}{1 - 3\sqrt{3}\xi_0} + B(1+z)^{3(1+\alpha)(1+d)(1-3\sqrt{3}\xi_0)} \right)^{\frac{1}{1+d}} \right)^{\frac{1}{1+\alpha}} \right]^{\frac{1}{2}} \\ &\quad \times \left[1 - \left(C + \left(\frac{1}{1 - 3\sqrt{3}\xi_0} + B(1+z)^{3(1+\alpha)(1+d)(1-3\sqrt{3}\xi_0)} \right)^{\frac{1}{1+d}} \right)^{\frac{1}{1+\alpha}} \right] \end{aligned}$$

$$\times \left[\frac{3\Omega_0 H_0^2}{\rho_1} \right]^{\frac{1}{2}}. \quad (29)$$

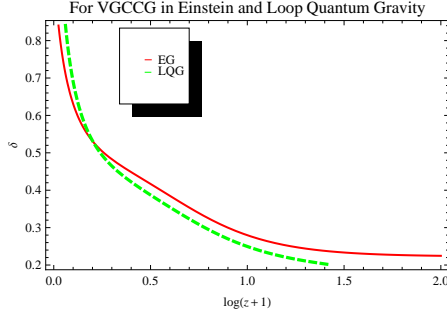


Figure 7: Plot of δ versus $\log(1+z)$.

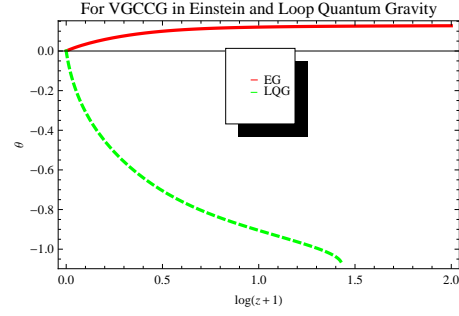


Figure 8: Plot of θ versus $\log(1+z)$.

For VGCCG in Einstein gravity and loop quantum cosmology, the plots of the time varying parameters δ , θ , c_e^2 , c_s^2 , ω_c and ω versus $\log(1+z)$ can be obtained by inserting the values of corresponding expression of H and shown in Figures 7-12. The constant parameters are same as chosen in previous section. It can be observed from Figure 7 that the evolution of density perturbation depicts decreasing behavior with the passage of time. However, the other perturbed quantity θ exhibits the same behavior as observed for VMVCG in Einstein and loop quantum cosmology (Figure 8). We can also see from Figure 9 and 10 that perturbed as well as usual squared speed of sound remains positive which leads to the stability of the model at the present as well as later epoch for Einstein/Loop quantum cosmology. The perturbed EoS (Figure 11) depicts the matter dominated era of the universe at initial as well as later epoch in Einstein and loop quantum cosmology. The usual EoS parameter shows the phantom-like universe at the present epoch as well as at the later epoch (Figure 12).

4 Conclusion

There exist various cosmological and astrophysical investigations for the various CG models. These models also used to investigate the problems of early universe such as inflationary era [45, 46, 47]. The accreting phenomenon onto

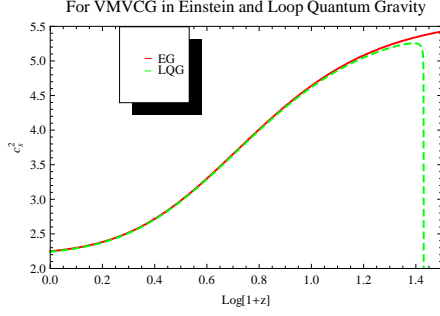


Figure 9: Plot of c_s^2 versus $\log(1+z)$.

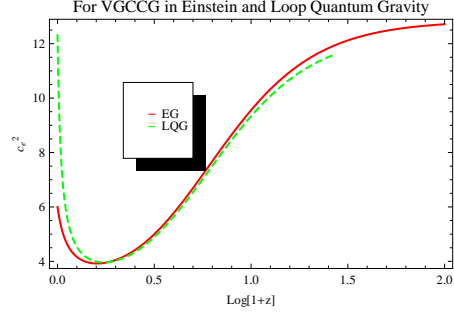


Figure 10: Plot of c_e^2 versus $\log(1+z)$.

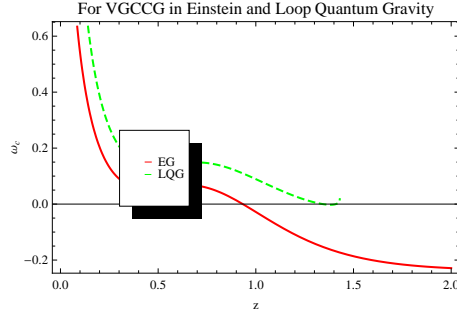


Figure 11: Plot of ω_c versus $\log(1+z)$.

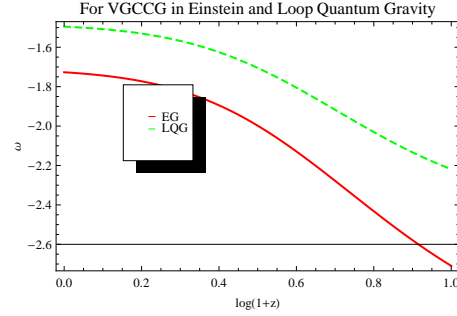


Figure 12: Plot of ω versus $\log(1+z)$.

various black holes has also been discussed with these models. It is found that the phantom-like behavior of CG models reduces the mass of black holes. Babichev et al. [48] argued that black hole loses its mass during the accretion of phantom DE onto it. In order to support this idea, different attempts have been made in the presence of different phantom-like DE models such as CG, GCG, MCG, VMCG models etc [49, 50]. Phantom-like behavior of CG is also more effective in the wormhole physics where the event horizon can be avoided due to its presence [51, 52].

Bhar et al. [53] investigated the new non-singular model for anisotropic charged fluid sphere in $(2+1)$ -dimensional anti de-Sitter spacetime corresponding to the exterior BTZ spacetime. They have chosen we choose modified Chaplygin gas in order to solve the Einstein-Maxwell field equations and show that their model satisfies all required physical conditions for representing compact stars. Sharif and Jawad [52] have constructed traversable,

asymptotically flat and stable wormhole solutions with the help of GCCG.

In the present work, we have focused on the top-hat collapse of a spherically symmetric fluid. We have explored the non-linear evolution of VMVCG and VGCCG perturbations in classical top-hat profile by taking the background of flat FRW metric. In the perturbed region, we have observed the natures of perturbed quantities like density contrast (δ) and θ , EoS parameter (ω_c), square speed of sound (c_e^2) for both chaplygin gas models in Einstein and loop quantum cosmology. We have also calculated the usual EoS parameter (ω), square speed of sound (c_s^2) in both scenarios. We have analyzed all the cosmological parameters graphically versus logarithmic scale of redshift parameters (Figures 1-12). We have summarized our results as follows

- **For VMVCG:**

The evolution of density perturbation exhibited the increasing behavior with the passage of time for both gravities. However, θ shows the increasing behavior with the passage of time in Einstein gravity while shows decreasing behavior in loop quantum cosmology (Figure 2). The usual as well as perturbed squared speed of sound have led the stability of VMVCG model in both gravities (Figures 3 and 4). Figure 5 (the perturbed EoS) has suggested the matter dominated era of the universe at initial epoch while goes to phantom era at the later epoch by crossing quintessence as well as Λ CDM limit. The usual EoS parameter shows phantom-like universe at the present epoch while goes towards quintessence region of the universe at the later epoch (Figure 6).

- **For VGCCG:**

It can be observed from Figure 7 that the evolution of density perturbation depicts decreasing behavior with the passage of time. However, the other perturbed quantity θ exhibits the same behavior as observed for VMVCG in Einstein and loop quantum cosmology (Figure 8). We have also observed that perturbed as well as usual squared speed of sound remains positive which leads to the stability of the model at the present as well as later epoch for Einstein/LQC (Figures 9 and 10). The perturbed EoS (Figure 11) depicts the matter dominated era of the universe at initial as well as later epoch in Einstein and loop quantum cosmology. The usual EoS parameter shows the phantom-like universe at the present epoch as well as at the later epoch (Figure 12).

Finally, it can be concluded that both chaplygin gas models support the

spherical collapse in Einstein as well as loop quantum cosmology because density contrast remains positive in both cases (Figures 1 and 7). Also, the perturbed EoS remains positive at the present epoch as well as near future (Figures 5 and 11) in both cases. It can be also remarked that some of our results are consistence with [9, 10, 14, 19, 20, 21].

Appendix

For the perturbed region, the basic equations are

$$\dot{\rho}_c = -3h(\rho_c + p_c), \quad \frac{\ddot{r}}{r} = \frac{-4\pi G(\rho_c + 3p_c)}{3}, \quad (30)$$

where r is the local scale factor and $h = \frac{\dot{r}}{r}$ relates to local expansion rate in the STHC framework

$$h = H + \frac{\theta}{3a}, \quad (31)$$

where $\theta = \nabla \cdot v$ is the divergence of the peculiar velocity v . The dynamical equations of density contrast δ and θ can be calculated as [14]

$$\delta' = -\frac{3}{a}(c_e^2 - \omega)\delta - [1 + \omega + (1 + c_e^2)\delta]\frac{\theta}{a^2 H}, \quad (32)$$

$$\theta' = -\frac{\theta}{a} - \frac{\theta^2}{3a^2 H} - \frac{3H}{2}(1 + c_e^2)\delta\Omega, \quad (33)$$

where $\Omega = \frac{\rho}{3H^2}$ and prime represents the derivative with respect to scale factor a . The above equations can be rewritten in terms of z as

$$\frac{d\delta}{dz} = \frac{3}{1+z}(c_e^2 - \omega)\delta - (1 + \omega + (1 + c_e^2)\delta)\frac{\theta}{H(z)} \quad (34)$$

$$\frac{d\theta}{dz} = \frac{\theta}{1+z} + \frac{\theta^2}{3H(z)} + \frac{3H(z)}{2(1+z)^2}(1 + 3c_e^2)\delta\Omega. \quad (35)$$

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